UConn **HCMI 4225** **Homework 3 Solutions**

1. A lottery costing \$1 is created based on a random drawing of a number between 1 and 10,000,000. If the player guesses the number, they win \$5,000,000. Suppose a person is risk neutral, i.e., their utility is linear in income ($U(X) = X$). Will this person ever play the lottery? Why would anyone ever play such a game?

   The expected increase in income from this lottery is given by: $E(X) = p_x * \$5,000,000 + (1 - p_x) * 0 = \frac{1}{10,000,000} * 5,000,000 = \frac{1}{2}$ Since utility is one-to-one with income, the expected increase in utility from this game will also be $\frac{1}{2}$. Since the game costs \$1, costing a utility of 1, the utility of keeping the money is greater than the utility of playing the game, so this person will not play. Some might, though, because they get utility from the game independent of the winnings or because they are risk loving.

2. Suppose that a utility function is $U(X) = \sqrt{X}$ where X is measured in thousands of dollars. Fred's current job pays \$2,500 per month with certainty. Fred can chose to work for himself and have a 50% chance of earning \$3,600 per month and a 50% chance of earning only \$1,600. Should Fred take the new job? Does your answer change if Fred's utility function is $U(X) = ln(X)$?

   Choosing to work for himself gives: $E(U(X)) = .5 * \sqrt{3,600} + .5 * \sqrt{1,600} = .5 * 60 + .5 * 40 = 30 + 20 = 50$ Fred's current job gives $U(2,500) = \sqrt{2,500} = 50$. So Fred is equally happy either way! Note that choosing to work for himself gives $E(X) = .5 * 3,600 + .5 * 1,600 = 1,800 + 800 = 2,600$.

   If the utility function is $ln(X)$, $E(U(X)) = .5 * \ln 3,600 + .5 * \ln 1,600 = .5 * 8.19 + .5 * 7.38 = 4.10 + 3.69 = 7.79$ Fred's current job gives $U(2,500) = \ln 2,500 = 7.82$. So Fred is happier staying at his current job.

3. Suppose that everyone has the same utility function and an annual income but people face different risks to health. Person A has a 10% chance of experiencing a health shock that requires \$100 in expenses while Person B has a 0.1% chance of experiencing a health shock that requires \$10,000. Calculate the expected loss of the two individuals. Graphically illustrate that Person B would be willing to pay a greater risk premium for insurance that Person A although the expected loss is the same for both types of people. Explain your answer. Expected Loss of A: $.1 * \$100 = \$1$. Expected Loss of B: $.001 * \$10,000 = \$1$.

   In your graph, even though the expected loss and the initial incomes are the same, the amount of income person B ends up with if they get sick is much lower, so the line for person B is below the line for person A. Thus the risk premium for B is greater than for A.

4. Fred has a job where he earns Y per year but there is a probability P Bob will be injured on his job. If injured, Fred will not be able to work and his income will fall to zero. Write an equation for Fred's expected utility in the absence of any type of insurance. a)What is the certainty equivalent? $U^{-1}(E(U(Y))) = U^{-1}((1-P)U(Y) + P*0) = U^{-1}((1-P)U(Y))$ Under workers' compensation, if a worker is injured on the job and unable to work, the workers' comp program will pay the worker a fraction θ of their income (Y) in the injured state. A premium, I, is taken from Fred's paycheck to fund this, so his income if he isn't injured is Y-I.

   b) What is his new expected income? Assume that if Fred is injured, he does not pay any premium.

   Assume that if Fred is injured, he does not pay any premium. $E(Y) = P\theta Y + (1-P)(Y-I)$

   c)If Fred is risk neutral, U(Y)=Y. In this case, what is his new expected utility?

   Assume that if Fred is injured, he does not pay any premium. $U(E(Y)) = E(Y) = P\theta Y + (1-P)(Y-I)$

   d)If Fred is risk averse, will his expected utility under this uncertainty be higher or lower?

   lower

   e) Solve for I assuming the insurance company makes an expected profit of 0

   The insurance company recieves $P\theta Y = (1-P)(I)$ $I = \frac{P}{1-P}\theta Y$ $I = Y(1 - \frac{P}{1-P}\theta)$

   f) What relationship do you expect between P, I and θ in the previous problem? Do you see that relationship in this problem? Increasing θ increases the amount Fred receives from the insurance company, so the premium I should increase if the insurance company is to make zero profit. Increasing P increases the probability the company will payout, so either I should increase, θ should decrease, or both if thecompany is to still make zero profit.