1. A lottery costing \$1 is created based on a random drawing of a number between 1 and 20,000,000. If the player guesses the number, they win \$15,000,000. Suppose a person is risk neutral, i.e., their utility is linear in income ($U(X) = X$). Will this person ever play the lottery? Why would anyone ever play such a game?

   The expected increase in income from this lottery is given by: $E(X) = p_x * \$15,000,000 + (1 - p_x) * 0 = \frac{1}{20,000,000} * 15,000,000 = \frac{3}{4}$ Since utility is one-to-one with income, the expected increase in utility from this game will also be $\frac{3}{4}$. Since the game costs \$1, costing a utility of 1, the utility of keeping the money is greater than the utility of playing the game, so this person will not play. Some might, though, because they get utility from the game independent of the winnings or because they are risk loving.

2. Suppose that a utility function is $U(X) = \sqrt{X}$ where X is measured in thousands of dollars. Franklin's current job pays \$2,500 per month with certainty. Franklin can chose to work for himself and have a 50% chance of earning \$3,600 per month and a 50% chance of earning only \$1,600. Should Franklin take the new job? Does your answer change if Franklin's utility function is $U(X) = ln(X)$?

   Choosing to work for himself gives: $E(U(X)) = .5 * \sqrt{3,600} + .5 * \sqrt{1,600} = .5 * 60 + .5 * 40 = 30 + 20 = 50$ Franklin's current job gives $U(2,500) = \sqrt{2,500} = 50$. So Franklin is equally happy either way! Note that choosing to work for himself gives $E(X) = .5 * 3,600 + .5 * 1,600 = 1,800 + 800 = 2,600$.

   If the utility function is $ln(X)$, $E(U(X)) = .5 * \ln 3,600 + .5 * \ln 1,600 = .5 * 8.19 + .5 * 7.38 = 4.10 + 3.69 = 7.79$ Franklin's current job gives $U(2,500) = \ln 2,500 = 7.82$. So Franklin is happier staying at his current job.

3. Suppose that everyone has the same utility function and an annual income but people face different risks to health. Person A has a 10% chance of experiencing a health shock that requires \$1000 in expenses while Person B has a 0.1% chance of experiencing a health shock that requires \$100,000. Calculate the expected loss of the two individuals. Which individual will be willing to pay more for insurance? Explain your answer.

   Expected Loss of A: $.1 * \$1000 = \$10$. Expected Loss of B: $.001 * \$100,000 = \$10$.

   One way to solve this problem is to graph the two situations. In your graph, even though the expected loss and the initial incomes are the same, the amount of income person B ends up with if they get sick is much lower, so the line for person B is below the line for person A. Thus the risk premium for B is greater than for A. An alternative approach is to consider the difference between an individual with risk-free income equal to the expected income of a risky individual but the same utility functions. We know that the risk-free individual will have higher utility. Similarly, an individual with lower risk will have more utility than an individual with higher risk, even if the two have the same utility functions.

4. Otto has a job where he earns $50,000 per year but there is a probability 2% Otto will be injured on his job. If injured, Otto will not be able to work and his income will fall to zero. Otto's utility function is $U(x) = 2\sqrt{x}$.
   a)What is Otto's expected earnings? What is his expected utility? What is the certainty equivalent? What is his risk premium?
   b)If Otto's employer gave him worker's compensation which paid him half of his salary in the case of injury, what is his new expected earnings? What is his new expected utility? What is the new certainty equivalent? What is his new risk premium? If the worker's compensation is through a third-party insurer, what is the insurance companies profit? What is the ratio of profit to capital if the insurance company must keep 10% of capital on hand?
   c) and d) Do parts a) and b) but now assume his utility function is $U(x) = \ln(X)$.