1. Sketch a plot of $U(x) = \sqrt{(x)}$, an agent's utility function. For this utility function, what is what is the utility of $x = 0, 1, 4, 9$?

   The Law Of Diminishing Marginal Utility states that all else equal as consumption increases the marginal utility derived from each additional unit declines. Does this utility function follow the law of diminishing marginal utility? How do you know?

2. What is the expected value of a lottery that pays $100 with probability 1% and $0 with probability 99%?

   Write down the general form to calculate the expected value of a lottery which pays payoff X_1 with probability P_1 and payoff X_2 with probability P_2.

3. Assume $U(x) = \sqrt{(x)}$. What is the utility of a payoff of $0? of $100?

   What is the expected utility of a lottery that pays $100 with probability 1% and $0 with probability 99%?

   Write down the general form to calculate the expected utility of a lottery which pays payoff X_1 with probability P_1 and payoff X_2 with probability P_2.

4. Expected utility is not the same as the utility of the expected value. What is the utility of the expected value of the lottery in the previous problems?

   Is this greater than or less than the expected utility?

5. Expected utility is usually less than expected value. There are intuitive reasons for this and mathematical reasons. Can you explain an intuitive reason?

(optional) Can you explain a mathematical reason?

6. A lottery costing $1 is created based on a random drawing of a number between 1 and 20,000,000. If the player guesses the number, they win $15,000,000. Suppose a person is risk neutral, i.e., their utility is linear in income ($U(X) = X$). Will this person ever play the lottery? Why would anyone ever play such a game?

7. Suppose that a utility function is $U(X) = \sqrt{X}$ where X is measured in thousands of dollars. Franklin's current job pays $2,500 per month with certainty. Franklin can chose to work for himself and have a 50% chance of earning $3,600 per month and a 50% chance of earning only $1,600. Should Franklin take the new job?

Does your answer change if Franklin's utility function is $U(X) = ln(X)$?

8. For our utility function, what values of x give $u(x) = 0, 1, 2, 3, 4$?

In general, we are interested in the inverse utility function. If $y = U(x) = \sqrt{(x)}$, what is the inverse utility function look like?

What is the inverse utility function for $y = U(x) = ln(x)$?

9. **Certainty equivalent** is the value of a sure outcome which gives the same utility as the expected utility of a lottery. What was the expected utility of the lottery we discussed earlier?

What is the inverse utility of that value?

This is the certainty equivalent of the lottery. An agent might be said to be indifferent between the choice of playing the lottery or getting the certainty equivalent of the lottery.

10. The certainty equivalent from the previous problem is less than the expected value of the lottery. What is the difference between these two values?

    This difference is called the **risk premium**. Write down an equation which compares the expected utility, the expected value, and the risk premium.

11. **Risk Premium** is the difference between the expected value of a gamble and the value of a sure outcome such that the utility of that outcome equals the expected utility of the gamble. What is the expected value, expected utility, certainty equivalent, and risk premium of the following lottery: 40% chance of earning $2500/month 60% change of $1600/month?

12. Otto has a job where he earns $50,000 per year but there is a probability 2% Otto will be injured on his job. If injured, Otto will not be able to work and his income will fall to zero. Otto's utility function is $U(x) = 2\sqrt{x}$.

    What is Otto's expected earnings? What is his expected utility? What is the certainty equivalent? What is his risk premium? What is his expected loss?

13. Actuarilly fair insurance is insurance where the premium equals the expected loss. What is Otto's expected loss? If Otto buys an insurance plan for this premium which pays him nothing if he isn't injured, but pays him his expected earnings if he is injured, what is Otto's new earnings minus premium?

    What is the insurance company's expected profit?

The maximum insurance premium Otto would be willing to pay is his expected loss plus the risk premium. Why?

What is the insurance company's expected profit if Otto pays this premium?

14. A private insurance company will seek to maximize profit while governments may tend towards actuarilly fair social insurance schemes. What are one or two pros and cons of private vs. public run insurance?

15. Suppose you have a \$200,000 home (wealth). There is a 2% chance that a fire will damage your house. If it does, will cause \$75,000 in loss (L). $U(W) = ln(W)$, wealth in good state is W and in bad state is $W - L$. Can you calculate your expected wealth, expected loss, expected utility, certainty equivalent, and risk premium?

16. If an insurance company charged the risk premium and insured your property, what is the company's expected profit in the previous problem?

If the company is required to keep 100% of the possible payment to you of \$75,000 on hand, what is the ratio of profit to capital on hand?

If you had \$75,000, would you be interested in this level of expected profit?

If the company were required to keep 5% of the possible payment on hand, what would be the ratio of expected profit to capital on hand?